An Approximate Analysis of Roll/Trim Dispersion

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An approximate approach for estimating roll/trim re-entry dispersion has been developed. Mean roll rate, spin variance, percent roll through zero, percent second encounter, and impact miss have been described by analytical expressions which permit the influence of design parameters to be rapidly assessed. This approximate analysis is suitable for guiding the development of empirical correlations based on existing flight data or Monte Carlo computations and for computation of relative performance in preliminary design studies where large numbers of configurations and trajectory points must be surveyed.

Nomenclature

A_B = base area = $\pi D_B^2/4$
A_T = instantaneous cross-sectional area of tip
C_{l_F} = frustum heat-shield roll moment coefficient
C_{l_T} = nosetip roll moment coefficient (referenced to tip dimensions)
C_{l_p} = frustum roll damping coefficient
C_{l_α} = vehicle lift derivative at zero incidence
C_{N_T} = nosetip normal force coefficient (referenced to tip dimensions)
CR = semiminor axis of impact ellipse = $|R(\bar{\rho}=1)|$
D_B = base diameter
DR = semimajor axis of impact ellipse = CR/sinγ_E
g_0 = acceleration of gravity
h = altitude
h_s = atmospheric scale height
I = roll moment of inertia
M_v = vehicle mass
$\bar{P}$ = nondimensional spin rate = P/P_0, where P_0 is the initial spin rate
R = complex envelope radius = $Z+iY$
R_N = nose radius
R_B = base radius
SM = instantaneous static margin
t = time
$\bar{V}$ = nondimensional velocity = V/V_E ($=e^{-\alpha'\bar{\rho}}$), where V_E is entry velocity
X_{cg} = axial length from stagnation point to vehicle center-of-gravity station
Y,Z = coordinates orthogonal to zero lift trajectory
α_T = trim angle
α' = $g_0 h_s \rho_s (2\beta \sin\gamma_E)^{-1}$
β = ballistic coefficient
γ_E = re-entry angle
$\Delta\bar{r}_{cg}$ = $\Delta r/D_B$, where Δr is radial center-of-gravity offset
$\bar{\rho}$ = nondimensional density = ρ_∞/ρ_s ($=e^{-h/h_s}$), where ρ_s is sea level ambient density
$\bar{\rho}_{NT}$ = nondimensional density for nosetip transition, (start of trim)
$\bar{\rho}_{TR}$ = nondimensional density for frustum transition, (start of roll perturbations)
$\sigma_{\bar{P}}$ = variance of nondimensional spin rate
ϕ = lift vector orientation angle in Y-Z plane

Received Oct. 19, 1977; revision received Aug. 15, 1978.
Index category: Entry Vehicle Dynamics and Control.
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Introduction

ROLL/TRIM dispersion results from lift nonaveraging produced by body-fixed vehicle asymmetries.[1-3] The problem is usually described in terms of a static trim resulting from asymmetric nosetip shape change.[4,5] Simultaneously, there are roll rate perturbations produced by heat-shield and nosetip ablation. Significant impact miss results when such trimmed vehicles experience transient zero or near-zero roll rate conditions.

The roll/trim dispersion problem is statistical in nature because the relevant aerodynamic error sources are themselves subject to randomness due to variabilities in material properties and angle of attack at deployment. For this reason, the standard computational approach for estimating roll/trim impact miss has been numerical trajectory simulation requiring a large number of re-entry trials, and furnishing an accurate numerical description of the impact miss statistics for each particular situation considered.[5]

This paper presents an approximate approach for estimation of roll/trim dispersion. This effort was motivated by a desire to provide a rapid computational tool, suitable for assessing the influence of various design parameters on vehicle performance, which could be easily incorporated into existing preliminary design codes. The specific objectives were to develop approximate expressions for the mean values and standard deviations of the spin rate and impact miss components. Identification of key "lumped parameter" groupings which display the design variables and which could be used for correlation of existing flight data or rigorous solution outputs was also desired.

Analysis

Trajectory Description

The lateral motions caused by lift nonaveraging will produce a spiraling motion about a mean flight path which diverges from the zero lift trajectory. The mean flight path can be pictured as defining an envelope about a straight (zero lift) line, neglecting curvature of the nonlifting trajectory for high ballistic coefficient bodies.[6] It follows from Newton's law applied to the lateral motion that

$$\frac{d^2R}{dt^2} = \frac{C_{l_\alpha}\alpha_T\rho_s V_E^2 A_B}{2M_v}\bar{\rho}\bar{V}^2 e^{i\phi} \tag{1}$$

Inherent in the derivation of Eq. (1) is the assumption that the lateral motion represents a small uncoupled perturbation superimposed upon the basic ballistic re-entry motion.

Integration can be most conveniently accomplished by transforming variables from t to $\bar{\rho}$ and by reversing the order of integration of the resulting double integral:

$$R(\bar{\rho}=1) = -\frac{\rho_s h_s^2 A_B}{2 M_v \sin^2\gamma_E}\int_{\bar{\rho}_{NT}}^{1} C_{l_\alpha}\alpha_T e^{i\phi}\ell n\bar{\rho}' F(\alpha',\bar{\rho}')\,\mathrm{d}\bar{\rho}' \tag{2}$$

where

$$F(\alpha',\bar{\rho}') = e^{-\alpha'\bar{\rho}'}\left\{1-(\ell n\bar{\rho}')^{-1}\sum_{j=1}^{\infty}\frac{(-\alpha')^j[1-(\bar{\rho}')^j]}{j\cdot j!}\right\}$$

From Eq. (2) it is clear that the impact miss integral, $R(\bar{\rho}=1)$, is a statistical parameter with randomness entering through the lift amplitude term $C_{l_\alpha}\alpha_T$ and the phase $e^{i\phi}$.

Spin Rate

Perturbing rolling moments produced by heat-shield and nosetip ablation are the primary sources for re-entry vehicle spin variations. Effects of roll damping and trim/center-of-gravity offset have also been considered. The angular momentum equation can be written as

$$\frac{\mathrm{d}\bar{P}}{\mathrm{d}t} = \frac{\rho_s V_E^2 A_B D_B}{2IP_0}\bar{\rho}\bar{V}^2\left\{C_{l_F} \pm C_{l_T}\left(\frac{R_N}{R_B}\right)^3 \pm C_{l_\alpha}\alpha_T\Delta\bar{r}_{cg} - C_{l_p}\bar{P}\bar{V}^{-1}\frac{P_0 D_B}{2V_E}\right\} \tag{3}$$

After performing the transformation of variables, it is noted that a first-order, linear, nonhomogenous, ordinary differential equation is obtained. The solution is

$$\bar{P} = \exp\left(-\int_{\bar{\rho}_{TR}}^{\bar{\rho}} f_1\mathrm{d}\bar{\rho}\right)\left\{1+\int_{\bar{\rho}_{TR}}^{\bar{\rho}} f_2\exp\left(\int f_1\mathrm{d}\bar{\rho}\right)\mathrm{d}\bar{\rho}\right\} \tag{4}$$

where

$$f_1 = (P_0 D_B/2V_E)\lambda C_{l_p}$$

$$f_2 = \lambda\bar{V}\{C_{l_F} \pm C_{l_T}(R_N/R_B)^3 \pm C_{l_\alpha}\alpha_T\Delta\bar{r}_{cg}\}$$

$$\lambda = \frac{h_s\rho_s V_E D_B A_B}{2IP_0\sin\gamma_E}$$

Clearly, the exponential term represents the effects of roll damping, while the other term represents the combined effects of the perturbing moment sources. The plus/minus signs have been explicitly displayed to indicate that nosetip torque and center-of-gravity offset have no preferred directionality. The (ensemble) re-entry vehicle spin rate is therefore determined only by the mean applied frustum torque and roll damping. It should be noted that the statistical roll rate is a linear sum of the various contributors (random variables). It should also be noted that the trim angle can be replaced in favor of the perturbing nosetip normal force by use of a simple moment balance expression:

$$(C_{l_\alpha}\alpha_T)(\mathrm{SM})A_B = C_{N_T}X_{cg}A_T \tag{5}$$

Dispersion Models

Model representations for the (statistical) frustum heat shield roll torque, nosetip roll torque, and nosetip normal force are key ingredients in assessing vehicle performance. However, a significant amount of uncertainty presently exists regarding specific modeling features. Therefore, it was simply assumed that each contributor could be modeled as the product of a known altitude- or density-dependent function and a non-altitude-dependent random variable which determined the level for the contributor of interest.

Thus, the dispersion models were assumed to be representable by the following forms:

$$C_{l_F} = g_1(\bar{\rho})C_1 \tag{6a}$$

$$C_{l_T} = g_2(\bar{\rho})C_2 \tag{6b}$$

$$C_{N_T} = g_3(\bar{\rho})C_3 \tag{6c}$$

where the $g_i(\bar{\rho})$ represent the altitude dependence of the various force/torque coefficients and the C_i are the fundamental random variables in the problem, i.e., the "level statistics." Generally, these g_i functions also include dependencies upon the design parameters and the secondary "onset" random variables $\bar{\rho}_{TR}$, $\bar{\rho}_{NT}$.

In order to understand how applicable empirical force models can be constructed from sets of flight or ground test experimental data, consider Fig. 1 of Ref. 4. The perturbing nosetip normal force is seen to vary monotonically with relative recession, eventually reaching a statistical plateau level. The required density-dependent function g_3 can be curve-fit from the nondimensional force coefficient, $C_{NT}/(C_{NT}$ plateau), vs relative recession with the corresponding dependence of relative recession upon density computed using an applicable nosetip response expression. The statistics for the plateau level C_3 can be simultaneously constructed from the observed data spread. It should be noted that the force models utilized in detailed numerical trajectory simulation codes are also generally constructed in this manner. These simpler force models will differ in the degree of smoothing applied to the functional forms and in the simplicity of the requisite support parameter descriptions. Lastly, it should also be clear that unity interaltitude correlation coefficients have been implicitly assumed as an intergral part of this simplified dispersion modeling.

Spin Rate Solution

Substitution of the exponential velocity law and the dispersion model representations into the formal spin rate solution results in

$$\bar{P} = e^{-AI_1(\bar{\rho})}\left\{1+\lambda\sum_{i=1}^{3}\alpha_i(\bar{\rho})C_i\right\} \tag{7}$$

and

$$\phi = \frac{P_0 h_s}{V_E\sin\gamma_E}\int_{\bar{\rho}_{TR}}^{\bar{\rho}}\bar{P}e^{\alpha'\bar{\rho}}\frac{\mathrm{d}\bar{\rho}}{\bar{\rho}} = \frac{P_0 h_s}{V_E\sin\gamma_E}\left[\beta_0(\bar{\rho})+\lambda\sum_{i=1}^{3}\beta_i(\bar{\rho})C_i\right] \tag{8}$$

where

$$A = \frac{P_0 D_B}{2V_E}\lambda\bar{C}_{l_p}$$

$$I_1(\bar{\rho}) = \int_{\bar{\rho}_{TR}}^{\bar{\rho}}\frac{C_{l_p}(\bar{\rho})}{\bar{C}_{l_p}}\mathrm{d}\bar{\rho}$$

$$\alpha_i(\bar{\rho}) = \int_{\bar{\rho}_{TR}}^{\bar{\rho}}\exp(AI_1(\bar{\rho}')-\alpha'\bar{\rho}')G_i(\bar{\rho}')\mathrm{d}\bar{\rho}'$$

$$G_1(\bar{\rho}) = g_1(\bar{\rho})$$

$$G_2(\bar{\rho}) = \pm(R_N/R_B)^3 g_2(\bar{\rho})$$

$$G_3(\bar{\rho}) = \pm\Delta\bar{r}_{cg}(X_{cg}A_T/(\mathrm{SM})A_B)g_3(\bar{\rho})$$

where the definitions of the β_i are apparent from Eqs. (7) and (8) and where $\bar{C}_{l_p}$ is an appropriate reference value for the

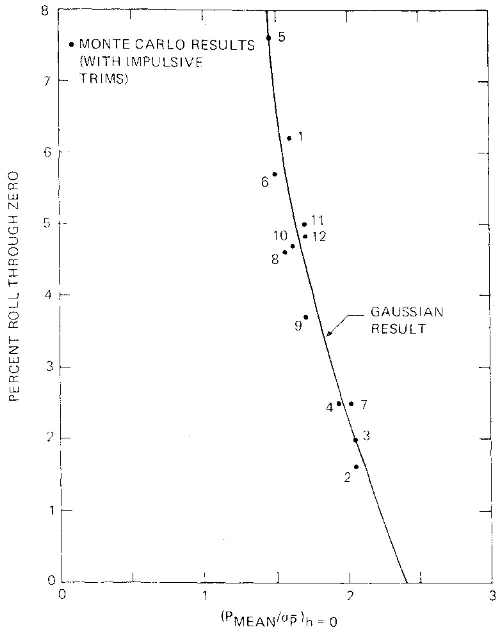

Fig. 1 Roll through zero probability.

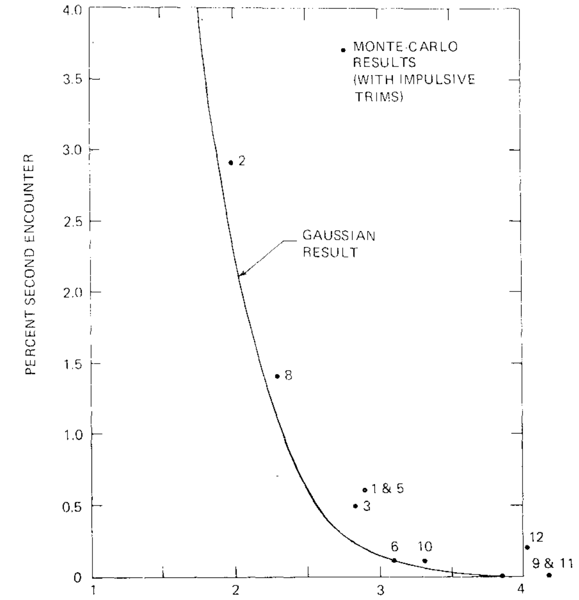

Fig. 2 Second encounter probability.

configuration and trajectory of interest.[7,8] The α_i represent altitude/configuration dependent influence coefficients for the various dispersion contributors. The parameter λ represents a global efficiency for spin production.

Spin Rate Statistics

As noted previously, the nosetip roll torque and center-of-gravity offset/trim term average out when computing the (ensemble) average spin rate:

$$(\bar P)_{\text{mean}} = e^{-AI_I(\bar\rho)}\{1+\lambda\alpha_I(\bar\rho)(C_I)_{\text{mean}}\} \tag{9}$$

Vehicle spin statistics are informative in regard to accuracy (percent roll through zero) and structural response (percent second encounter). In order to analytically estimate these properties, first it is necessary to compute the spin rate variances:

$$\sigma_{\bar P}^2 = [\bar P^2]_{\text{mean}} - [(\bar P)_{\text{mean}}]^2 \tag{10}$$

The variance can be evaluated by substitution of Eqs. (7) and (9) into Eq. (10). A Taylor expansion technique is required in order to account for the contributions due to the onset variables, $\bar\rho_{TR}$, $\bar\rho_{NT}$. The result shows that the spin variance is basically a sum of contributions due to the level statistics, with the α_i serving as weighting factors. An additional, typically small, contribution from the onset variables is also included.

Approximations for percent roll through zero and percent second encounter were constructed by employing the following reasoning. Focusing attention on those cases which exhibit a tendency to despin (roll down), it is clear that the spin variance line which cuts the axis $\bar P = 0$ at impact determines percent roll through zero. That is, percent roll through zero should be dependent upon the number of $\sigma_{\bar P}$'s which exist between the mean spin rate and $\bar P = 0$ at impact $\bar\rho = 1$. For example, if the -1σ spin variance line crossed the axis at impact, a larger value of percent roll through zero would be anticipated than if a -3σ crossing had occurred. Furthermore, since Gaussian representations have typically been applied to the various dispersion contributors, it is expected from the linearity of the roll rate equation that the relevant spin distribution function should also be Gaussian. The expected result should then correspond to the area under the normal curve of error outboard of the position $(\bar P_{\text{mean}}/\sigma_{\bar P})_{\bar\rho=1}$. That is,

$$\begin{matrix}\text{percent roll}\\ \text{through zero}\end{matrix} = 100\int_{(\bar P_{\text{mean}}/\sigma_{\bar P})_{\bar\rho=1}}^{\infty} E(r)\,\mathrm{d}r \tag{11}$$

where

$$E(r) = (1/\sqrt{2\pi}\,)e^{-r^2/2} \tag{12}$$

Figure 1 presents a comparison of this simple expression with results provided from detailed Monte Carlo trajectory simulations for a variety of typical re-entry cases. It can be seen that an acceptable prediction is furnished provided the ratio $\bar P_{\text{mean}}/\sigma_{\bar P}$ can be estimated accurately enough.

Figure 2 presents the corresponding result for percent second encounter:

$$\begin{matrix}\text{percent second}\\ \text{encounter}\end{matrix} = 100\int_{[(\bar P_{\text{crit}}-\bar P_{\text{mean}})/\sigma_{\bar P}]_{\bar\rho=1}}^{\infty} E(r)\,\mathrm{d}r \tag{13}$$

where the number of $\sigma_{\bar P}$'s between the resonant frequency $\bar P_{\text{crit}}$ and the mean spin rate is the relevant index.

Several points should be discussed before proceeding. First, the Gaussian nature of these typical spin rate distributional properties results from the linear nature of the governing equation combined with the present tendency to use essentially Gaussian dispersion contributors. It appears that this behavior can best be discerned after a suitable parameter (such as $\bar P/\sigma_{\bar P}$) is selected and has not been previously identified. Second, this simple approximation for the spin statistics would require modification if significantly non-Gaussian dispersion contributors were to be specified. In this regard, it should be noted that for each particular replication of a Monte Carlo trajectory reconstruction, the roll rate results from the combined effects of a relatively small number of random variables. Therefore, the central limit theorem,[9] which applies asymptotically as the number of random variables becomes large, does not imply that the distributions obtained from large numbers of multitrial Monte Carlo calculations should, in general, be Gaussian. Third, presently available Monte Carlo results indicate that dispersion contributions due to second encounter are generally not significant due to the large spin rates present, and have, therefore, been neglected for this approximate analysis.

Impact Miss Distance

Returning to Eq. (2) and explicitly representing the lift term using Eq. (5), it results that

$$\mathrm{DR} = \left| -\frac{h_s^2 \rho_s A_{T_0} X_{cg} C_3}{2 M_v \sin^3 \gamma_E \mathrm{SM}_0} \int_0^1 F_1(\xi) e^{i\phi(\xi)} \mathrm{d}\xi \right| \tag{14}$$

where

$$F_1(\xi) = g_3(\xi)(A_T/A_{T_0})(\mathrm{SM}/\mathrm{SM}_0)^{-1}$$

$$\{\ln[\bar{\rho}_{NT} + (1-\bar{\rho}_{NT})\xi]\} F(\alpha',\xi)(1-\bar{\rho}_{NT})$$

$$\xi = \frac{\bar{\rho} - \bar{\rho}_{NT}}{1-\bar{\rho}_{NT}}$$

and where the subscript 0 denotes an initial value.

It is noted that ξ arises from constructing a variable of order unity for the density regime over which trim exists. The terms appearing in $F_1(\xi)$ can all be given physical meanings. The first term indicates how large the nosetip normal force coefficient has become (relative to its plateau level). The second term represents the effect of blunting due to nosetip recession. The third term reflects the effects of static margin variations during re-entry. The fourth term represents the time remaining for an instantaneous lateral velocity to produce impact miss (dispersion). The last term comes from the transformation to ξ. Similarly, many of the terms in the coefficient can be interpreted physically, i.e., X_{cg} relates to moment arm for nosetip force.

Consideration of the form of $F_1(\xi)$ provides the information that a fairly regular behavior exists. That is, $F_1(\xi)$ is 0 at $\xi = 0$ since no trim has built up. Similarly, $F_1(\xi)$ is 0 at $\xi = 1$ since no travel time is left. A maximum is typically exhibited somewhere within the interval, depending on the form of g_3 selected. Thus, by selecting an appropriate reference value for F_1, an essentially unit function can be defined. The downrange dispersion can then be written as

$$\mathrm{DR} = \mathrm{CC} \left| \frac{C_3}{(C_3)_{\mathrm{mean}}} \right| \left| \int_0^1 Y_1(\xi) e^{i\phi(\xi)} \mathrm{d}\xi \right| \tag{15}$$

where

$$Y_1 = F_1(\xi)/F_1(\xi_{\mathrm{ref}})$$

and where the "correlation coefficient" (CC) is defined by

$$\mathrm{CC} = -\frac{h_s^2 \rho_s A_{T_0} X_{cg} (C_3)_{\mathrm{mean}}}{2 M_v \sin^3 \gamma_E \mathrm{SM}_0} F_1(\xi_{\mathrm{ref}})$$

It is noted that the coefficient (CC) basically reflects the normal force "potential" of the given situation. The remaining integral term reflects the "effectiveness" or likelihood that this potential will be exercised. Since identification of a possible correlation format for displaying existing flight data or rigorous solution outputs was also an objective of this task, this reasoning was pursued one step further. It appeared reasonable to assume that a correlation of nondimensional impact miss (i.e., "miss"/CC) vs percent roll through zero (i.e., "likelihood") might exist. Figure 3 presents results of typical Monte Carlo trajectory simulations presented in this correlation format. Here $\hat{\mathrm{DR}}$ is the standard deviation in the downrange impact coordinate, whereas DR is the semimajor axis. The mean values of the impact miss coordinates can be envisioned as corresponding to the mean roll behavior and so are usually negligible because of lift averaging.

A linear behavior appears to exist. However, significant scatter is clearly evident. Some portion of this scatter may be present because CC does not contain all the vehicle dependency; α' is still present in evaluating the integral term in Eq. (15). However, it was demonstrated that significant scatter can be caused by attempting to describe a statistical distribution using only a single parameter. To clarify this point, consider the three data points: 23.1-23.3. These points were generated by using one given set of input data and performing three separate 1000 trial runs. Close inspection of the detailed results indicates that small numbers of re-entry trials at large impact miss distances (i.e., "tails") are responsible for this variation in $\hat{\mathrm{DR}}$. However, inspection of the cumulative distributions indicates that essentially identical results are being generated. Combining all three cases into an equivalent 3000 trial run yields results which are in good agreement with the linear fit.

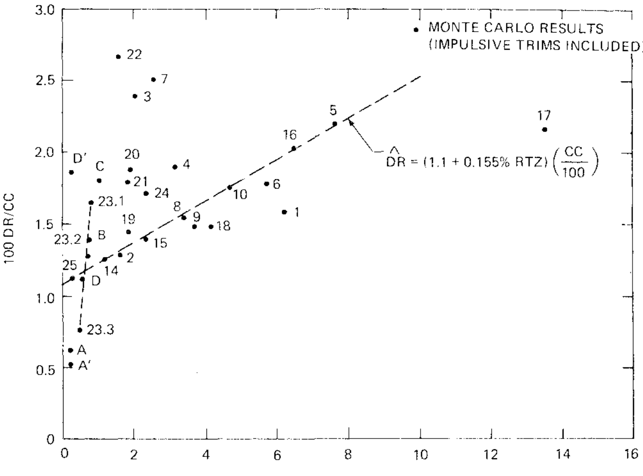

Fig. 3 Dispersion correlation.

With the feasibility of developing a correlation format demonstrated, direct solution to the dispersion integral, Eq. (15), was pursued. The expression was approximated as

$$\mathrm{DR} = C_R' \left| \sum_{K=1}^{N} U_K e^{i\phi_{K_0}} \int_0^1 Y_1(\xi') e^{i\phi_{K_1}\xi'} \mathrm{d}\xi' \right| \tag{16}$$

where

$$C_R' = \frac{\mathrm{CC}}{(1-\bar{\rho}_{NT})} \left| \frac{C_3}{C_{3_{\mathrm{mean}}}} \right|$$

Here the total interval in $\bar{\rho}$ has been partitioned into segments in order that the function ϕ has a satisfactory linear representation in each interval: $\bar{\rho}_K \leq \bar{\rho} \leq \bar{\rho}_{K+1}$. This was achieved by first mapping $(\bar{\rho}_K, \bar{\rho}_{K+1})$ into (0,1) by means of the transformation

$$\bar{\rho} = U_K \xi' + \bar{\rho}_K$$

where

$$U_K = \bar{\rho}_{K+1} - \bar{\rho}_K$$

This transformation allows the convergence properties of shifted Chebyshev polynomials to be exploited.[10] For a given interval, an expansion in these polynomials yields the smallest error over the entire interval for a fixed degree m. This error is obtained by evaluating the $(m+1)$st coefficient of the shifted Chebyshev polynomial. The intervals were chosen so that the error in the segmented linear representation of ϕ was satisfactorily small.

In general, the functions $\alpha_1(\xi')$ and $Y_1(\xi')$ must also be expanded using Chebyshev methods:

$$\alpha_1(\xi') = \sum_{j=0}^{L} \alpha_{jK1} \xi'^{j}$$

$$Y_I(\xi') = \sum_{j=0}^{m} f_{jK}\xi'^{j}$$

for $\bar{\rho}_K \leq \bar{\rho} \leq \bar{\rho}_{K+I}$.

The dispersion integral is thus reduced to an easily integrated approximate form:

$$\mathrm{DR} = C_R' \left| \sum_{K=I}^{N} U_K e^{i\phi_{K_0}} \int_0^I e^{i\phi_{K_I}\xi'} \sum_{j=0}^{m} f_{jK}\xi'^{j} \mathrm{d}\xi' \right| \quad (17)$$

The coefficients ϕ_{K_0}, ϕ_{K_I} involve linear combinations of the primary random variables ("level" statistics): C_I-C_3 and $\Delta\bar{r}_{cg}$. The f_{jK} and α_{jKi} are functions of only the secondary (onset) statistics $\bar{\rho}_{TR}$, $\bar{\rho}_{NT}$. To obtain first- and second-order impact miss statistics, DR can be developed as a series about the expected values of the input random variables:

$$E[\mathrm{DR}] \doteq \mathrm{DR}_0 + \frac{I}{2}\left[\sum_{i=I}^{6} \sigma_i^2 \frac{\partial^2(\mathrm{DR})_0}{\partial y_i^2}\right] \quad (18)$$

$$E[\mathrm{DR}^2] \doteq \mathrm{DR}_0^2 + \sum_{i=I}^{6}\left[\mathrm{DR}_0 \frac{\partial^2(\mathrm{DR})_0}{\partial y_i^2} + \left(\frac{\partial(\mathrm{DR})_0}{\partial y_i}\right)^2\right]\sigma_i^2 \quad (19)$$

$$\mathrm{Var}[\mathrm{DR}] \doteq E[\mathrm{DR}^2] - E^2[\mathrm{DR}] \quad (20)$$

where $y_I = C_I$, $y_2 = C_2$, $y_3 = C_3$, $y_4 = \Delta\bar{r}_{cg}$, $y_5 = \bar{\rho}_{TR}$, and $y_6 = \bar{\rho}_{NT}$ and where σ_i^2 is the corresponding variance of y_i. It has also been assumed that the errors are uncorrelated ($E[y_i y_j] = 0$ for $i \neq j$). It should be noted that the impact miss distribution function can be further approximated by continuation of this procedure to higher orders.[9]

Conclusion

An approximate approach for estimating roll/trim re-entry dispersion has been developed based on standard statistical theory methodology. That is, whenever a given result can be formally expressed in terms of a set of random variables whose individual statistical behaviors are characterized, then the statistical behavior of the result is also determined. Statistical roll behavior is found to be particularly amenable to such analytical description because of the linearity of the governing equations.

Results include the identification of the Gaussian nature of typical present-day computed impact spin rate statistics when properly parameterized and the decomposition of the impact miss distance into identifiable "normal force potential" and "effectiveness" (or "likelihood") components. In retrospect, neither of these results is surprising, given the generally assumed Gaussian nature of the primary input random variable statistics and the physical processes operative in producing roll/trim dispersion. It also appears that the "lumped-parameter" groupings identified in the present analysis do provide convenient formats for the development of empirical correlations based on available flight data or rigorous solution outputs.

Results obtained for one particular set of sample representations of the error source models indicate that typical cases can be run at very low cost. For card input, the compilation times were about 40 seconds and the execution times were about 5 seconds on an IBM 370/145 computer. Comparison of computed parameters from this approximate method with Monte Carlo computation results indicates that mean spin rate can be computed to accuracy levels of about 10-15%, while statistical properties such as percent roll through zero and standard deviation of downrange impact miss are computed to accuracy levels from 25-50%, depending on the particular parameter and the particular case considered.

Acknowledgment

This work was supported by the Space and Missile Systems Organization of the Air Force Systems Command under Contract F04701-76-C-0173.

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